# Analysis and Design of Three Transmission Zeros Bandpass Filter Utilizing Triple-Mode Dielectric Loaded Cubical Cavity

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*Abstract* — **An equivalent circuit for the third-order triple-mode cross-coupled asymmetric response filter is proposed which includes the direct-coupling between the input and output ports. Based on this circuit, the existence of three transmission zeros for the filter is established through mathematical expressions derived for the computation of the zeros. To verify the accuracy of the theoretical transmission zeros, a triple-mode cross-coupled asymmetric response filter with a centre frequency at 1.705 GHz, is designed and implemented, using a dielectric loaded cubical cavity resonator. Very close agreement between the computed and measured results of the filter is achieved.**

## I. Introduction

Because of their temperature stability, low loss and compact size, dielectric resonators are finding increasing applications in high-performance narrow-band bandpass filters for modern microwave communication systems, especially in satellite and mobile communications [1]-[2]. To reduce the size of the dielectric loaded cavity resonator filter, the multi-mode technique has been employed [3]-[4]. With cross-couplings between nonadjacent resonances, transmission zeros are introduced in the stopband of the filter, which help to improve the skirt characteristics, thus resulting in further miniaturization of the bandpass filter through reduction of the number of resonators or cavities [5]-[6].

A vast number of publications have been devoted to dual-mode cavity filters, while there is less attention in the literature on triple-mode cavity filters [7]-[9]. In particular, it appears that the only explanation of attenuation poles in dual-mode cavity filters has been presented by Zaki [10] and Awai [11]. Although Hong and Lancaster [12] has reported the performance of a microstrip cross-coupled trisection bandpass filter with asymmetric frequency characteristics, no analysis was given to explain the occurrence of the extra attenuation poles in the stopband.

In this paper, we show the origin of the transmission zeros that appear in the triple-mode cavity resonator filter through an equivalent-circuit model of the structure. It is shown that the filter response can have three transmission zeros in the stopband. Mathematical expressions are derived to calculate the transmission zero frequencies. For illustration, a triple-mode cross-coupled asymmetric response bandpass filter, is designed and implemented using a dielectric loaded cubical cavity resonator operating in the $TM_{11\delta}$ degenerate modes. For this filter, the theoretical and experimental responses are compared, thus verifying the accuracy of the theoretical expressions for the computation of the transmission zeros.

## II. Transmission Zeros of Canonical Triple-mode Cross-coupled Filter

An equivalent circuit of the 3[rd] order triple-mode cross-coupled filter inclusive of the input and output direct coupling is shown in Fig. 1.

The circuit along the main path of the filter, indicated by the dashed-box in Fig. 1, represents the cross-coupled asymmetric triple-mode filter with a cross-coupling between the first and third resonators. This cross-coupling between the resonators will result in a triple-mode filter with an asymmetric frequency response having a transmission zero of finite frequency on either the high side or the low side of the passband depending on the signs of the admittance inverters $J_{12}$, $J_{23}$ and $J_{13}$.

The chain (or ABCD) matrix of the circuit in the dashed-box can be derived using a conventional technique:

$$\begin{bmatrix} \dfrac{J_{34}^2\left(P_1P_2-J_{12}^2\right)}{J_{01}J_{34}\left(J_{13}P_2-J_{12}J_{23}\right)} & \dfrac{j\left(P_1P_2P_3-J_{23}^2P_1-J_{23}^2P_2-J_{12}^2P_3+2J_{12}J_{23}J_{13}\right)}{J_{01}J_{34}\left(J_{13}P_2-J_{12}J_{23}\right)} \\ \dfrac{-jJ_{01}^2J_{34}^2P_2}{J_{01}J_{34}\left(J_{13}P_2-J_{12}J_{23}\right)} & \dfrac{J_{01}^2\left(P_2P_3-J_{23}^2\right)}{J_{01}J_{34}\left(J_{13}P_2-J_{12}J_{23}\right)} \end{bmatrix} \quad (1)$$

where

$P_i=b_i\left(\dfrac{\omega}{\omega_{oi}}-\dfrac{\omega_{oi}}{\omega}\right)=\dfrac{C_{Li}}{\Delta}\left(\dfrac{\omega}{\omega_o}-\dfrac{\omega_o}{\omega}\right)+B_{Li}=\dfrac{C_{Li}}{\Delta}\left(\lambda+X_i\right)$ evaluated at $\omega_o$,

in which

$$\omega_{oi}=\frac{1}{\sqrt{L_iC_i}}=\omega_o\sqrt{1-\frac{B_{Li}}{C_{Li}/\Delta+B_{Li}/2}},$$

and



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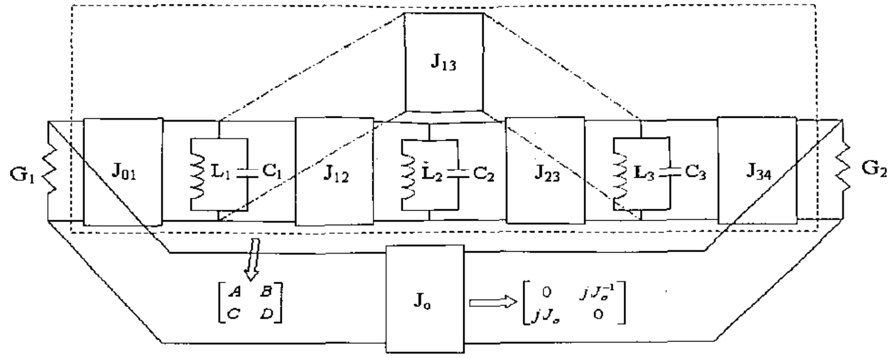


Fig. 1. Equivalent circuit of 3rd order triple-mode cross-coupled filter.

$$b_i = \omega_{oi} C_i = \frac{\omega_{oi}}{\omega_o}\left(\frac{C_{Li}}{\Delta} + \frac{B_{Li}}{2}\right), \text{ for } i=1,2,3.$$

In the previous expressions, $\Delta = BW/f_o$, is the fractional bandwidth, $\lambda = (\omega/\omega_o - \omega_o/\omega)$, is the normalised fractional bandwidth,

$$X_i = \frac{B_{Li}\Delta}{C_{Li}} = 2\left(\frac{\omega_o}{\omega_{oi}} - \frac{\omega_{oi}}{\omega_o}\right)\left(\frac{\omega_o}{\omega_{oi}} + \frac{\omega_{oi}}{\omega_o}\right)^{-1},$$

and

$$k_{12} = \frac{J_{12}}{\sqrt{b_1 b_2}},\ k_{23} = \frac{J_{23}}{\sqrt{b_2 b_3}},\ k_{13} = \frac{J_{13}}{\sqrt{b_1 b_3}}$$

are the coupling coefficients. For a filter terminating in 50Ω ($G_1 = G_2 = G_o = 0.02$), then $J_{01} = J_{34} = 0.1414$, and $Q_{ei} = b_1$ is the input external Q-factor whereas $Q_{eo} = b_3$ is the output external Q-factor.

In (1), $C_{Li}$ ($i=1, 2, 3$) denotes the capacitance and $B_{Li}$ denotes the frequency-invariant susceptance of the lowpass prototype filter with a normalized termination of unity. These unknown lowpass element values may be determined by a method described by Hunter [13] or through an optimisation process. It is observed from (1) that each resonator is, in general, not synchronously tuned to the filter's centre frequency $f_o$, and $X_i$ is a constant reactance representing the offset from the bandpass filter's centre frequency, for narrow bandpass filter design. The transmission coefficient of the cross-coupled asymmetric bandpass filter can be easily expressed in terms of the matrix elements, $A$, $B$, $C$, and $D$, given by,

$$S_{21} = \frac{2\sqrt{G_1 G_2}}{G_1 G_2 B + G_1 A + G_2 D + C} \quad (2)$$

Focusing on the numerator of $S_{21}$ reveals that it can be expressed as a linear polynomial given by,

$$N_{21} = 2J_{01}J_{34}\sqrt{G_1G_2}\,(J_{13}P_2 - J_{12}J_{23}) = \alpha\left(\lambda + X_2 - \frac{J_{12}J_{23}\Delta}{J_{13}C_{L2}}\right) \quad (3)$$

where $\alpha = (2J_{01}J_{13}J_{34}C_{L2}\sqrt{G_1G_2})/\Delta$

If $\lambda_z$ is the value of $\lambda$ when $N_{21}=0$, then the location of the transmission zero of the bandpass filter is,

$$f_z = \left(0.5\lambda_z + \sqrt{1 + 0.25\lambda_z^2}\right) f_o, \quad (4)$$

found easily by solving an appropriate quadratic equation.

For illustration, a 3rd order cross-coupled bandpass filter with centre frequency $f_o = 1.705$ GHz, fractional bandwidth $\Delta = 0.03$ and return loss of –20 dB with a transmission zero occurring at the lowpass prototype normalized frequency of 2 is synthesized. The element values of the lowpass prototype are found to be [13]: $C_{L1} = C_{L3} = 0.85220$, $B_{L1} = B_{L3} = 0.11647$, $C_{L2} = 1.41625$, $B_{L2} = -0.76659$, $J_{12} = -J_{23} = 1.0$, and $J_{13} = -0.48405$. Substituting these values into (1), leads to $f_{o1} = f_{o3} = 1.7015$ GHz, $f_{o2} = 1.7189$ GHz, $Q_{ei} = Q_{eo} = b_1 = b_3 = 28.407$, $b_2 = 47.207$, $k_{12} = 27.308\text{x}10^{-3}$, $k_{23} = -27.308\text{x}10^{-3}$, and $k_{13} = -17.040\text{x}10^{-3}$. With $X_2 = -16.238\text{x}10^{-3}$, (3) can be solved to find the transmission zero of the filter, yielding $\lambda_z = 59.999\text{x}10^{-3}$, and $f_z = 1.757$ GHz. Simulation of this bandpass filter is performed using ADS™ [14] leading to the response depicted in Fig. 2. As indicated by marker m4 in Fig. 2, the response of the cross-coupled bandpass filter has a single transmission zero occurring at the upper stopband since $J_{12}J_{23}/J_{13}$ is positive. On the other hand, the transmission zero will appear at the lower stopband if $J_{12}J_{23}/J_{13}$ is negative.

To compute the bandpass filter response inclusive of the effect of $J_o$, $S_{21}$ has to be derived using the chain matrices of the dashed-box and the admittance-inverter $J_o$ shown in Fig.1. In this case the numerator is found to be,

$$N_{21} = 2G_oJ_o\left[P_1P_2P_3 - J_{23}^2P_1 - \left(J_{13}^2 - \frac{J_{01}J_{34}J_{13}}{J_o}\right)P_2 - J_{12}^2P_3 + J_{12}J_{23}\left(2J_{13} - \frac{J_{01}J_{34}}{J_o}\right)\right]$$

$$= 2G_oJ_ob_1b_2b_3\left[\lambda_1\lambda_2\lambda_3 - k_{23}^2\lambda_1 - \left(k_{13}^2 - \frac{G_ok_{13}}{J_o\sqrt{Q_{ei}Q_{eo}}}\right)\lambda_2 - k_{12}^2\lambda_3 + k_{12}k_{23}\left(2k_{13} - \frac{G_o}{J_o\sqrt{Q_{ei}Q_{eo}}}\right)\right] \quad (5)$$

where $\lambda_i = P_i/b_i = d_i(\lambda + X_i)$ and $d_i = C_{Li}/(b_i\Delta) = 0.5\left(\frac{\omega_o}{\omega_{oi}} + \frac{\omega_{oi}}{\omega_o}\right) \approx 1.0$ for $i = 1, 2, 3.$

The transmission zeros of the bandpass can be obtained by solving $N_{21} = 0$, which is a cubic equation in the form,

$$a_o\lambda^3+3a_1\lambda^2+3a_2\lambda+a_3=0 \quad (6)$$

where $a_o=1$, $a_1=(X_1+X_2+X_3)/3$,

$$a_2=\frac{1}{3}\left[X_1X_2+X_2X_3+X_3X_1-\frac{k_{23}^2}{d_2d_3}-\frac{k_{13}}{d_1d_3}\left(k_{13}-\frac{G_o}{J_o\sqrt{Q_{ei}Q_{eo}}}\right)-\frac{k_{12}^2}{d_1d_2}\right]$$

$$a_3=X_1X_2X_3-\frac{k_{23}^2X_1}{d_2d_3}-\frac{k_{13}X_2}{d_1d_3}\left(k_{13}-\frac{G_o}{J_o\sqrt{Q_{ei}Q_{eo}}}\right)-\frac{k_{12}^2X_3}{d_1d_2}+\frac{k_{12}k_{13}}{d_1d_2d_3}\left(2k_{13}-\frac{G_o}{J_o\sqrt{Q_{ei}Q_{eo}}}\right)$$

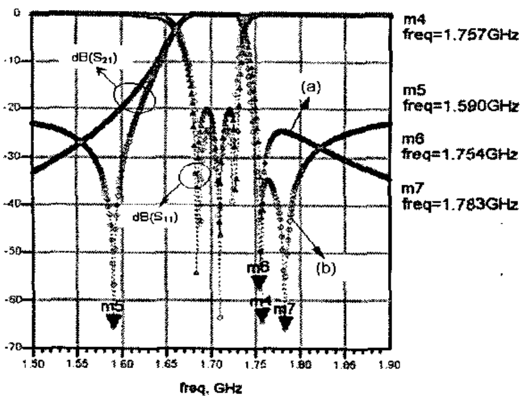


Fig. 2. (a) Cross-coupled asymmetric response with a single transmission zero, and (b) the response with three transmission zeros, of the bandpass filter.

Putting $a_o\lambda=y-a_1$, the resulting equation in y can be reduced to,

$$y^3+3Ny+M=0 \quad (7)$$

where

$$M=a_o^2a_3-3a_oa_1a_2+2a_1^3 \quad and \quad N=a_oa_2-a_1^2$$

Depending on the value of $R=M^2+4N^3$, up to three real roots are possible for the case of the cross-coupled filter with the effect of direct-ports-coupling taken into consideration. Assuming $J_o$=0.96x10$^{-3}$ and the external quality factors $Q_{ei}=Q_{eo}=Q_e$=28.407 computed as earlier, a circuit simulation using ADS™ is performed for the filter in Fig. 1. Circuit tuning was performed to reduce the resonant frequency of the second resonator for a return loss better then –20 dB over the passband, and coupling coefficients are also tuned to yield three distinct transmission zeros in the simulated response as depicted in Fig. 2. The circuit parameters obtained from the simulated bandpass filter are $f_{o1}=f_{o3}$=1.7015 GHz, $f_{o2}$=1.7177 GHz, $Q_{ei}=Q_{eo}=b_1=b_3=Q_e$=28.407, $b_2$=47.173, $k_{12}$=25.678x10$^{-3}$, $k_{23}$=-25.678x10$^{-3}$, and $k_{13}$=-18.446x10$^{-3}$. From these element values, the constant reactances are computed to be $X_1=X_3$=4.1098x10$^{-3}$, $X_2$=-14.842x10$^{-3}$. Parameters $M$ and $N$ are found to be 5.3123x10$^{-4}$ and -5.1023x10$^{-3}$ respectively with $a_1$=-2.2075x10$^{-3}$, $a_2$=-5.0974x10$^{-3}$ and $a_3$=5.6501x10$^{-4}$ from (6) and (7). Upon substitution of values of $M$ and $N$ into $R$, we have $R$=-2.4912x10$^{-7}$ and therefore (7) has three distinct real roots which can be expressed in terms of the trigonometric functions [15]. The roots of the cubic equation in (7), in this case, are given by,

$$y_z=\left\{2r\cos\frac{1}{3}\theta,\ 2r\cos\frac{1}{3}(\theta+2\pi),\ 2r\cos\frac{1}{3}(\theta+4\pi)\right\} \quad (8)$$

where $r=\sqrt{-N}$ and $\theta=\cos^{-1}\left(\frac{-M}{2\sqrt{-N^3}}\right)$

Substituting $M$ and $N$ into (8) the three roots of (7) are $y_z$={-0.13837, 0.038406, 0.099963}. The corresponding $\lambda_z$'s are $\lambda_z=y_z-a_1$={-0.13616, 0.04061, 0.10217} and hence the three transmission zero frequencies are computed to be 1.593 GHz, 1.740 GHz and 1.794 GHz respectively, which are all well within the error of 0.8% as compared to the corresponding simulated transmission zeros as indicated by markers m5 to m7 in Fig. 2. The slight deviation of the computed transmission zero frequencies from the simulated results is attributed to the approximation used during the analysis in representing the offset of the resonant frequency of each resonator from the filter centre frequency by the constant reactance $X_i$'s.

## III. TRIPLE-MODE FILTER IMPLEMENTATION

As an illustration, a triple-mode cross-coupled bandpass filter based on the parameters obtained from the circuit simulation in ADS™ is designed and implemented using a dielectric loaded cubical cavity resonator operating in $TM_{11\delta}$ degenerate modes [16]. A cubical cavity of side 80mm is loaded symmetrically with a cubical dielectric of side 29.4mm and relative permittivity of 37.0, Fig. 3. The cylindrical conductors fixed to the 50Ω SMA connectors at the input/output realize the desired external quality factor, $Q_e$=28.407. The three coupling screws A, B and C are utilized to realize the filter of asymmetric response with the three single transmission zeros. Locations of coupling screws A, B and C, and the magnitude of $J_o$ are determined by two-port measurement as presented by Zaki [10] and Awai [11]. The coupling coefficients realized in this case are $k_{12}=-k_{23}$=25.678x10$^{-3}$ and $k_{13}$=-18.446x10$^{-3}$. The value of $J_o$ is measured to be 0.96x10$^{-3}$ as used in the analysis. Three tuning screws are also added to tune the resonant frequency of each degeneracy independently as shown in Fig. 3.

The measured performance of the filter is depicted in Fig. 4, together with the computed response from ADS™. Very close agreement between the computed and measured transmission zero frequencies of the cross-coupled filter is obtained as can be seen in Fig. 4.

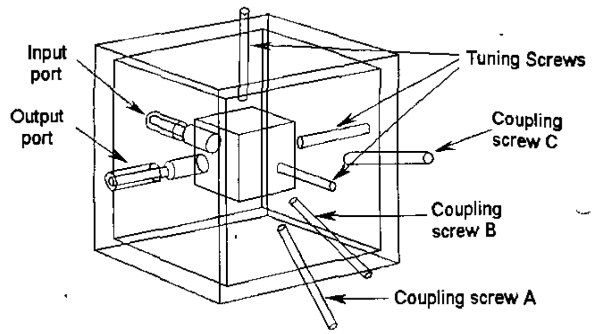


Fig. 3. The 3rd order triple-mode cross-coupled asymmetric response bandpass filter.

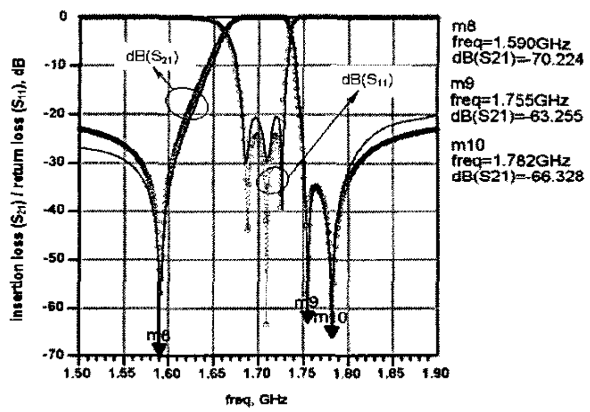


Fig. 4. (a) Measured response (——), and (b) computed response (--o--) of the triple-mode cross-coupled asymmetric bandpass filter with three transmission zeros in the stopband.

## IV. Conclusion

An equivalent circuit for the triple-mode cavity resonator filter including the coupling between the input and the output was proposed. Based on this equivalent circuit, the existence of three transmission zeros in the filter response was successfully demonstrated. Mathematical expressions were derived for the computation of the transmission zeros. With the aid of these expressions, a triple-mode bandpass filter using a dielectric loaded cubical cavity resonator operating in the $TM_{11\delta}$ degenerate modes was designed and implemented to meet the transmission zeros and the asymmetric response of a filter specification. Very close agreement between the simulated and measured performances of the filter was obtained.

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